

*An Account of two Books.*I. *SAGGI di NATURALI ESPERIENZE fatte nell' ACADEMIA del CIMENTO, in FIRENZE, A. 1667. in Fol.*

**T**His Book was lately by two excellent persons of the *Florentine Virtuosi*, viz. *Lorenzo Magalotti*, and *Paulo Falconieri*, presented to the *Royal Society*, in the name of His Highness Prince *Leopold of Tuscany*, that great Patron to real Philosophy. The Book contains these particulars:

1. An explication of the Instruments, employed in these Experiments.
2. Exp. belonging to the natural pressure of the Air.
3. Exp. concerning artificial Conglaciations.
4. Exp. about natural Ice.
5. Exp. about the change of the capacity of Metal and Glass.
6. Exp. touching the compression of Water.
7. Exp. to prove that there is no positive Lightness.
8. Exp. about the Magnet.
9. Exp. about Amber, and other substances of a vertue Electrical.
10. Exp. about some changes of Colours in divers Fluids.
11. Exp. touching the motions of Sound.
12. Exp. concerning Projectils.
13. Various Experiments.

As all these Heads are very considerable, and of main importance to Philosophy, so doubtless will the handling of them be by competent Judges found worthy of these famous *Academians del Cimento*.

II. *VERA CIRCULI ET HYPERBOLÆ QUADRATURA, in propria sua proportionis specie inventa & demonstrata, à JAC. GREGORIO SCOTO, Patavii, in 4o.*

**T**His Tract perused by some very able and judicious Mathematicians, and particularly by the Lord Viscount *Brouncker*, and the Reverend Dr. *John wallis*, receiveth the Character of being very ingeniously and very Mathematically written, and well worthy the study of Men addicted to that

that Science : that in it the *Author* hath delivered a new Method Analytical for giving the Aggregate of an Infinite or Indefinite converging *Series* : and that from that ground he teaches a Method of *Squaring the Circle, Ellipsis, and Hyperbola*, by an Infinite Series, thence calculating the true dimensions as near as you please : And lastly, that by the same method from the *Hyperbola* he calculateth both the *Logarithms* of any *Natural Number* assign'd, and *vice versa*, the *Natural Number* of any *Logarithm* given.

Only a few of these Books were printed by the *Author* for his own use, and that of his Friends, and a Copy sent over whereby to reprint it here, which is now a doing.

*The Mathematical Mr. John Collins, upon a more particular examination of this Book, communicated what follows concerning the same.*

The *Author's* Computation of the *Area* of a *Circle* agrees with the Numbers of *Van Ceulen*; and his computation of the supplemental spaces between the *Hyperbola* and its *Asymptote* by Parallels to the other *Asymptote*, is correspondent to what *Gregory* of *St. Vincent* and his Commentators *Francis Aynscomb* and *Alphonse de Sarasa* have demonstrated concerning the *Logarithms*, as represented by those spaces, *viz.* That if one *Asymptote* be divided into a rank of continual Proportionals, and if parallels to the other *Asymptote* be drawn passing through the said rank, and be terminated at the *Hyperbola*, the spaces contain'd between each such pair of Parallels, are equal to each other, and so added or conceived to be one continued space, may represent the *Logarithms*; or the said Proportionals, fitted in parallel to the divided *Asymptote*, do the like, by reason that a *Rectangle* apply'd to the several Terms of a *Geometrical Progression* increasing, renders another in the same *Ratio* decreasing. And both performed by the above-mentioned Analytical method of conveying complicated *Polygons* circumscrib'd and inscrib'd in the sector of a *Circle, Ellipsis, or Hyperbola*, which he asserts to be quantities like *Surds*, not absolutely to be express'd in Numbers.

And it being manifest, that the making of the *Table* of *Logarithms* is in effect the same thing as the computing of *Area's* of those supplemental spaces, the *Author* accordingly applies it thereto

thereto, and finds the *Logarithms* of all *Primitive Numbers* under 1000 by one Multiplication, two Divisions, and the Extraction of the Square Root; but for *Prime Numbers* greater, much more easily.

Concerning the construction of *Logarithms*, Mr. *Nicholas Mercator* hath a Treatise, intituled *Logarithmotechnia*, likewise at the *Press*. from which the *Reader* may receive further satisfaction. And as for *Primitive Numbers*, and whether any odd number proposed less than 100000 be such, the *Reader* will meet with a satisfactory *Table* at the end of a Book of *Algebra*, written in *High Dutch* by *John Henry Rohn*, now translated and enriched, and near ready for publick view.

*The Area of an Hyperbola not being yet given by any Man, we think fit a little to explain the Author's meaning.*

In *Figure 1.* Let the Curve *DIL* represent an *Hyperbola*, whose *Asymptotes* *AO*, *AK*, make the Right Angle *OAK*, the Author propounds to find the *Hyperbolick* space *ILNK*, contained by the *Hyperbolical* Line *IL*, the *Asymptote* *KM*, and the two Right Lines *IK*, *LM*, which are parallel to the other *Asymptote* *AO*.

He puts the Lines  $IK = 1\ 000\ 000\ 000\ 000$   
 $LM = 1\ 000\ 000\ 000\ 000\ 0$   
 $AM = 1\ 000\ 000\ 000\ 000$   
 Hence  $KM = 9\ 000\ 000\ 000\ 000$

Whence he finds the space *L I K M*

to be 5230 258 509 299 404 562 401 78681 too little.  
 2230 258 509 299 404 562 401 78704 too great.

*Note*: If *IK* be put for an *Unit*, then *LM* may represent 10, and *HG* 1000, and *FE* 1024: And by what is demonstrated by *Gregory* of *St. Vincent*, it holds,

As the space *IBLMKI*, Is to the Logarithm of *LM*, to wit, of 10: So is the space *IBEFKI*, To the Logarithm of the Number represented by the Line *EF*, to wit, of 1024.

The *Author* by the same method finds the *Area* of the space *GEFH* to be 237 165 266 173 160 421 183 067, and the space *L I K M* above-said being taken for the Logarithm of 10, and tripled, is the Logarithm of 1000, the which added to the space now found, makes the sum 693 147 18055994529141719170, and 1024, being the 10th Power of 2, the 10th part of this number is the *Hyperbolical Logarithm* of the Number 2, to wit, 693 147 1805599452914171917. And it holds by proportion,

As 23025850929940456240178700, the Logarithm of 10, To 693 147 1805599452914171917, the correspondent Logarithm of 2: So 1 000 000 000 000 000 000 000 000 000 000 000 0, the Logarithm of 10 in the  
 Tables,

Tables, To 3010299956639811952405804, the Logarithm of 2 in the Tables.

By this means the *Area* of one *Hyperbola* being computed, the *Area's* of all others may be thence argued, as is shewed by *Greg. St. Vincent*, and *Van Schooten* in *Traſtatu de Organica Conicarum ſeſtionum deſcriptione*.

If the Logarithm of 1 be put 0; and of 10, 1,000000: If between 1 and 10 you conceive 999999 mean Proportionals interjeſted; the firſt is 1,00000023025853.

If the Logarithm of 1 be put 0; and of 10, 100000: If you conceive 99999 mean Proportionals between 1 and 10, the firſt is 1,000023025853; if an infinite rank of theſe be continued, there is no number propoſed, but will go nigh to be found in this rank, and the number of Terms, by which it is removed from *Unit*, is the Logarithm of the Number ſo found. The *Ratio* of 1, to 1,00000023025853, ſome call *Elementum Logarithmicum*. See *Cavallieri's Trigonometry*.

The *Area* of an *Hyperbola* is frequently required in *Gauging*; as admit it were required to compute the Solidity of the *Segment* of an upright *Cone* cut by a *Plain*, that would cut the produced oppoſite *Cone*; in any ſuch *Cafe* the *Section* is an *Hyperbola*. But we will only take the *Instance*, when it is parallel to the *Axis*.

In *Figure II.* 1. Let BVA repreſent ſuch a *Cone*, VC its *Axis*, BSAR the *Circle* in the *Base*. And firſt, ſuppoſe this *Cone* cut by a *Plain* paſſing through the *Vertex* and the *Base* U S R U; then is the whole *Cone* divided into ſuch proportions as the *Area* of the *Circle* in the *Base*. Whence we diſcover the *uſe* and the *want* of a good Table of *Area's* of *Segments*; the beſt of which kind yet extant is in *Sibrand Hantz* his *Century of Geometrical Problems*, tranſlated out of *Dutch* into *English* by Captain *Thomas Rudd*, who omitted the ſaid Table; uſeful likewise for finding the *Area* of the *Segment* of an *Ellipſis*, and the obtaining the quantity of *Liquor* out of, or left in a *Cask* part empty.

And we hint, that a Table of *Natural Verſed Sines* is to be found in *Maginus*, and of *Logarithmical* ones in *Cavallieri's Directorium Univerſale Uranometricum*.

2. The former *Plain* did cut out a *Chord-line* in the *Base*, to wit, SR; through the ſame imagine another *Plain* to paſs, and to cut the *Cone* beneath the *Vertex*, as at O; then is the *Wedge* contain'd between both theſe *Plains* (to wit, VSR OV) equal to  $\frac{1}{3}$  of that *Cylindrick* or *Prifmatick* *Figure*, whoſe *Altitude* is equal to the *Perpendicular* VP falling from the *Vertex* of the *Cone* to the cutting *Plain*, and whoſe *Base* SORTS is the *Area* of the *Figure* cut; in this *cave*, an *Hyperbola*: When the *Plain* paſſeth parallel to the ſide BV, a *Parabola*; when it will meet with VB produced, a *Portion* of an *Ellipſis*. By this means, if a *Brewer's Tun* (taken to be a *Circular Truncus Coni*) lean, and be not cover'd over with *Liquor* in its bottom, it may be computed by ſubtracting the two known before-men-

tion'd

tion'd parts out of the whole : If it stand upright, and be divided by an *upright Plain* into two *Partitions*, imagine it to be a whole *Cone*, and first, by the method above, find the *Segment*, as of the whole, and afterwards of the additional *Top-Cone*, the difference of those two gives the Content of the correspondent *Partition*.

3. But if the *Liquor* cut both sides, the *Tun* leaning, as *BCDE*, in *Figure III.* suppose *BAE* to be the *Triangle* through the *Axis* of the whole *Cone*, then the *Elliptick Cone* *ACD* to the whole *ABE* is in a *Triplicate Ratio* of the *Side-line* *AB* or *AE*, to the *Geometrical Mean* between *AC* and *AD*, that is,

*As the Cube of the Side-line AB, is to the Solidity of the whole Cone ABE : So is the Cube of the Geometrical Mean between AC and AD, To the Solidity of the Elliptick Cone ACD.*

And this readily follows from the *Doctrine of Viviani de Maximis & Minimis*, where 'tis demonstrated, that any such *Elliptick Cones*, cut out of an *Upright Cone*, that have the *Area's* of their *Triangles* through the *Axis* equal, are equal to each other; and likewise to that *Upright Cone* which hath the same *Area* on its *Triangle* through the *Axis* on the former *Plain* thereof; and these *Area's* he calls their right *Canons*.

And the mean *Proportional* by 23. E. 6. finds the sides of an *Isosceles Triangle* in the *Plain* of the *Axis* equal to the *Scalene Triangle*; and then these *Cones* are to each other in a *Triplicate Ratio* of their *Axes*, *Side-lines*, or *Base-lines*, which are proportional to their *Axes*.

The *Area* of an *Hyperbola* being obtain'd, the *Solidity* of the *Hyperbolical Fusa* or *Spindles* (made by the rotation of an *Hyperbola* about its *Base*) and their *Trunci* are computed, according to *Cavallieri* (in his *Geometrical Exercises*, printed at *Bononia* 1647.) and the solid *Zones* of these *Figures* may be well taken to represent a *Cask*.

In the *SAVOR*,

Printed by T. N. for John Martyn, Printer to the Royal Society, and are to be sold at the Bell a little without Temple-Bar, 1667.

Fig: I

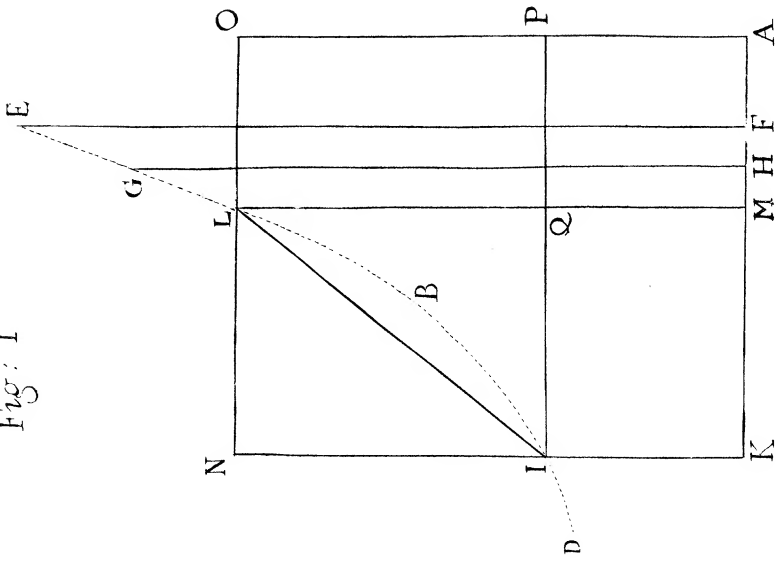


Fig: II.

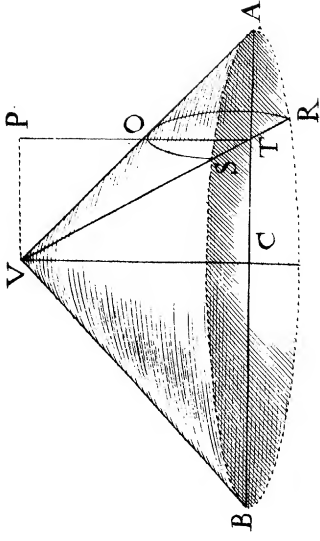


Fig: III

